C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS **SEMESTER V**

U15MMA503 – COMPLEX ANALYSIS

Time: Three Hours Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. If $z = x + i\,y$, then find the real and imaginary parts of $f(z) = z^2$.
2. Define entire function.
3. Define conformal transformation.
4. What is called a critical point of a transformation?
5. State Cauchy-Goursat theorem.
6. Define a simple arc.
7. State Taylor's theorem.
8. Write the Maclaurin's series of $f(z) = e^z$.
9. Define Residue.
10. Evaluate $\int_c e^z dz$ where $c: |z| = 1$.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) Prove that real and imaginary parts of an analytic function are harmonic.

(Or)

b) Derive Cauchy-Riemann equations in polar coordinates.

12. a) Show that the transformation $w = e^z$ maps the rectangular region $a \le x \le b$, $c \le y \le d$ onto the region $e^a \le \rho \le e^b$, $c \le \phi \le d$.

(Or)

b) Explain the function $z^{1/2}$.

13. a) Find $I = \int_C \bar{z}\, dz$ when C is the right-hand half $z = 2e^{i\theta}$ of the circle $|z| = 2$ from $z = -2i$ to $z = 2i$.

(Or)

b) State and prove the maximum modulus principle.

14. a) Find the Maclaurin series expansion of z^2e^{3z}.

(Or)

b) Expand the function $f(z) = (1 + z^2) / (z^3 + z^5)$ into a series involving powers of z.

15. a) Find the residues of $f(z) = (z + 1) / (z^2 + 9)$ at its poles.

(Or)

b) Find the residues of $f(z) = \cot z$ at its poles.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. Prove that if $f(z) = u(x, y) + v(x, y)$ and if $f'(z)$ exists at a point $z_0 = x_0 + y_0$, then the first order partial derivatives of u and v must exist at (x_0, y_0) and they must satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$ at (x_0, y_0). Also prove that $f'(z_0) = u_x + iv_x$ where these partial derivatives are evaluated at (x_0, y_0).
17. Discuss the function $1/z$.
18. State and prove Cauchy's integral formula.
19. State and prove Laurent's theorem.
20. Show that $\int_0^\infty \frac{2x^2-1}{x^4+5x^2+4}\, dx = \frac{\pi}{4}$.

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