C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS **SEMESTER V**

U15MMA502 / U14MMA502 – REAL ANALYSIS - I

Time: Three Hours Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. Define Equivalent Sets.
2. Define Sequence and Subsequence.
3. Explain Monotone sequence with an example.
4. Define Cauchy Sequence.
5. Is $\sum_{n=1}^{\infty} \frac{n}{n+1}$ convergent?
6. For what value of p does the series $1 / 1^p - 1 / 2^p + 1 / 3^p - 1 / 4^p + \ldots\ldots$ converge?
7. Define the class l^2.
8. Define Metric space M.
9. Define Continuous function.
10. Define dense with an example.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) If $A_1, A_2 \ldots\ldots$ are countable sets, then prove than $\bigcup_{n=1}^{\infty} A_n$ is countable.

(Or)

b) Prove that all subsequence's of a convergent sequence of real numbers converges to the same limit.

12. a) If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

(Or)

b) If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real number, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

13. a) State and prove Comparison test.

(Or)

b) If $\sum_{n=1}^{\infty} |a_n| = \infty$ and if $\lim_{n\to\infty} |a_n| / |b_n|$ exists, then prove that $\sum_{n=1}^{\infty} |b_n| = \infty$.

14. a) If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges, then prove that $\sum_{n=1}^{\infty} a_n$ converges.

(Or)

b) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then prove that $f(x) + g(x)$ has a limit as $x \to a$ and,in fact, $\lim_{x\to a} [f(x) + g(x)] = L + M$.

15. a) If the real valued functions f and g are continuous at $a \in R^1$, then prove that f+g ,f-g and fg are continuous at a. If $g(a) \neq 0$, then prove that f /g is also continuous at a.

(Or)

b) Let G be an open subset of the Metric space M. then prove that $G^1 = M - G$ is closed. Conversely, if F is a closed subset of M, then prove that $F^1 = M - F$ is open.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. If A is any non empty subset of R that is bounded below, then prove that A has a greatest lower bound in R.
17. Prove that the sequence $\{(1+1/n)^n\}_{n=1}^{\infty}$ is convergent.
18. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.
19. Let $<M, \rho>$ be a metric space. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points of M, then prove that $\{s_n\}_{n=1}^{\infty}$ is Cauchy.
20. Prove that the real valued function f is continuous at a iff wherever $\{x_n\}_{n=1}^{\infty}$ converges to a then $\{f(x_n)\}_{n=1}^{\infty}$ converges to $f(a)$.

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