C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS **SEMESTER V**

U15MMA501 / U14MMA501 – ABSTRACT ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. Define abelian group.
2. What is congruent?
3. Define homomorphism.
4. Prove that every subgroup of an abelian group is normal.
5. What is meant by even permutation?
6. Find the orbits and cycles of the following permutation
$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}$.
7. What is field?
8. Find the zero divisors of Z_{12}.
9. Define principal ideal ring.
10. Define prime element.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) State and prove Euler theorem.

(Or)

b) If G is a finite group whose order is a prime number p, then prove that G is a cyclic group.

12. a) Prove that HK is a subgroup of G if and only if HK = KH.

(Or)

b) If ϕ is a homomorphism of G into $\bar{G}$, then prove that.

(i) $\phi(e) = \bar{e}$, the unit element of $\bar{G}$.

(ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$

13. a) Prove that every permutation is the product of its cycles.

(Or)

b) Express (1, 2, 3), (4, 5) (1, 6,7,8,9) (1, 5) as the product of disjoint cycles.

14. a) If ϕ is a homomorphism of R into R' with Kernel $I(\phi)$, then prove that

(i) $I(\phi)$ is a subgroup of R under addition.

(ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.

(Or)

b Show that a finite integral domain is a field.

15. a) Prove that a Euclidean ring possesses a unit element.

(Or)

b) Let R be a Euclidean ring and a, b $\in$R. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. Prove that the relation $a \equiv b \bmod H$ is an equivalence relation.
17. State and prove Fundamental theorem of homomorphism.
18. Show that every group is isomorphic to a sub group of A(s) for some appropriate S.
19. If R is a ring, for all $a, b \in R$, then prove that.

    (i) $a0 = 0a = 0$

    (ii) $a(-b) = (-a)b = -(ab)$

    (iii) $(-a)(-b) = ab$.

    If, in addition, R has a unit element 1, then prove that

    (iv) $(-1)a = -a$

    (v) $(-1)(-1) = 1$

20. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

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