

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS

SEMESTER I

U15MMA102 – DIFFERENTIAL CALCULUS AND 2 - D GEOMETRY

Time: Three Hours

Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. Find the n^{th} derivative of $y = \cos(ax + b)$.
2. Find the Jacobian determinant for the transformation $u = x + 3y$ and $v = x - 3y$.

3. Show that in the curve $r = e^{\theta \cot \alpha}$, the polar subtangent is $r \tan \alpha$.
4. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1, 1)$.
5. Show that the asymptotes of $x^2 y^2 = c^2 (x^2 + y^2)$ are the sides of a square.
6. Write the formula to find the values of 'c' in finding the equation of asymptote of the curve $y = mx + c$.
7. Define the pole of the locus.
8. What are conjugate points and conjugate lines?
9. Write down the equation of the hyperbola conjugate to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
10. Find the conjugate diameter of $y = \frac{b}{a} \sin \theta$ with respect to the hyperbola.

SECTION - B (5 X 5 = 25 Marks)

Answer ALL Questions.

11. a) Find the n^{th} derivative of $\log(ax + b)$.
(Or)
12. b) Solve the total differential equation $y^2 dx - z dy + y dz = 0$.
c) Find the angle of intersection of the cardioid $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
(Or)
13. b) From the polar equation of the parabola, show that $p^2 = ar$.
c) Find the asymptotes of the cubic $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.
(Or)
14. b) Find the asymptotes of $y^3 = x(4 - x^2)$.
c) Find the equation of the chord of the circle $x^2 + y^2 - 11x - 2y - 24 = 0$ for which $(1, 2)$ is the middle point.
(Or)
15. b) If the polar of P passes through Q then show that the polar of Q passes through P.
c) If e_1, e_2 are the eccentricities of a hyperbola and its conjugate then show that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.
(Or)
16. b) If a pair of conjugate diameters meet the hyperbola and its conjugate in P and Q, then prove that $CP^2 - CQ^2 = a^2 - b^2$.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u .
17. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.
18. Find the asymptotes of $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$.
19. Find the equation of the polar of the point (x_1, y_1) with respect to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.
20. Prove that the acute angle between two conjugate diameters of an ellipse is a minimum when they are equal.
