C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS   **SEMESTER V**

U15EMA501 – GRAPH THEORY (ELECTIVE – I)

Time: Three Hours   Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. Define a bipartite graph.
2. Define a spanning subgraph.
3. Define incidence matrix of a graph.
4. Define union of two graphs.
5. What is connectivity of a graph?
6. Define a block.
7. Define a Hamiltonian cycle.
8. Define centre of a tree.
9. Define planar graph.
10. Define chromatic number.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

(Or)

    b) Show that $\alpha + \beta = p$.

12. a) Prove that a closed walk of odd length contains a cycle.

(Or)

    b) Prove that in a graph G, any $u - v$ walk contains a $u - v$ path.

13. a) Prove that every non-trivial connected graph has at least two points which are not cut points.

(Or)

    b) Prove that there is no 3-connected graph with 7 edges.

14. a) Prove that if G is a graph in which the degree of every vertex is at least two then G contains a cycle.

(Or)

    b) Prove that every connected graph has a spanning tree.

15. a) In any connected plane (p, q) graph $(p \geq 3)$ with r faces, prove that $q \geq 3r/2$ and $q \leq 3p - 6$.

(Or)

    b) Show that every k-chromatic graph has at least k vertices of degree at least $k - 1$.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. Prove that $\alpha' + \beta' = p$.

17. If A is a adjacent matrix of a graph with $V = \{v_1, \ldots, v_p\}$, prove that for any $n \geq 1$ the (i, j)th entry of A^n is the number of $v_i - v_j$ walks of length n in G.

18. Prove that a graph G with at least two points is bipartite iff all its cycles are of even length.

19. Prove that the following statements are equivalent for a connected graph G.

    (i) G is Eulerian.

    (ii) Every point of G has even degree.

    (iii) The set of edges of G can be partitioned into cycles.

20. Prove that $\chi'(K_n) = n$ if n is odd $(n \neq 1)$ and $\chi'(K_n) = n - 1$ if n is even.

****