C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS **SEMESTER V**

U14MMA503 – COMPLEX ANALYSIS - I

Time: Three Hours Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. What is a complex number?
2. Find the principal value of $(-i)^i$.
3. State the condition for a function "f" to be continuous.
4. Write Cauchy – Riemann equations in polar form.
5. Verify whether the function $u = y^3 - 3x^2y$ is harmonic or not.
6. Define conformal mapping.
7. Find the transformation of the function $w = \frac{4z^2}{(1-z)^2}$ at $z = \infty$.
8. What do you mean by a fixed point of a transformation?
9. Define a contour.
10. State Cauchy Goursat's theorem.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) Discuss some algebraic properties of complex numbers.

(Or)

b) Derive the exponential laws (i) $\exp(z_1 + z_2) = (\exp z_1)(expz_2)$ (ii) $\exp(z_1 - z_2) = \frac{(\exp z_1)}{(expz_2)}$.

12. a) Show that the function $\bar{z}$ is nowhere differentiable.

(Or)

b) Prove that the function $1 + z^2$ is entire.

13. a) Find $f(z)$ if $u = x^2 - y^2$.

(Or)

b) Show that the function $u(x, y) = \sin x \cos hy$ is harmonic. Find its harmonic conjugate and analytic function.

14. a) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$.

(Or)

b) Show that the transformation $w = z^2$ maps the rectangular hyperbolas onto the straight lines.

15. a) Find the value of the integral $\int_{C_1} z^2\, dz$ where $c_1: |z| = 1$.

(Or)

b) Evaluate $\int_c \frac{zdz}{(z-1)(z-2)}$ where $c: |z| = 3$.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. Explain stereographic projection.
17. State and prove the sufficient conditions for the existence of the derivative $f'(z)$.
18. Prove that the mapping $w = f(z)$ is conformal in D if $f(z)$ is analytic in a region D and if $f'(z) \neq 0$ in D.
19. Discuss the transformation $w = \sin z$.
20. State and prove Cauchy's Integral formula.

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