C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

B.Sc., MATHEMATICS **SEMESTER V**

U14EMA501 – GRAPH THEORY (ELECTIVE – I)

Time: Three Hours Maximum: 75 Marks

SECTION - A (10 X 2 = 20 Marks)

Answer **ALL** Questions.

1. Draw Peterson graph.
2. Draw the complete graph with 5 points.
3. Define incidence matrix of a graph.
4. Define union of two graphs.
5. Define terminal point.
6. Define bridge of a graph.
7. Define a Hamiltonian cycle.
8. Define connectivity of a graph.
9. Define tree.
10. Define centre of a tree.

SECTION - B (5 X 5 = 25 Marks)

Answer **ALL** Questions.

11. a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

(Or)

b) Prove that a set S⊆V is an independent set of G if and only if V-S is a covering of G.

12. a) Draw the graph K_5 and write down its incidence matrix.

(Or)

b) Prove that every graph is an intersection graph.

13. a) If G is not connected then $\bar{G}$ is connected.

(Or)

b) Prove that every non-trivial connected graph has at least two points which are not cutpoints.

14. a) Prove that if G is a k-connected graph the $q \geq \frac{pk}{2}$.

(Or)

b) Show that the Petersen graph is Non-Hamiltonian.

15. a) If G is a acyclic with p = q + 1 then prove that G is a tree.

(Or)

b) Prove that every connected graph has a spanning tree.

SECTION - C (3 X10 = 30 Marks)

Answer **ANY THREE** Questions.

16. Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

17. Let G_1 be a (p_1,q_1) graph and G_2 a (p_2,q_2) graph. Then show that.

    (i) $G_1 \cup G_2$ is a $(p_1+p_2,\ q_1+q_2)$ graph.

    (ii) G_1+G_2 is a $(p_1+p_2,\ q_1+q_2+p_1p_2)$ graph

18. Prove that a graph G with at least two points is bipartite iff all its cycles are of even length.

19. Prove that the following statements are equivalent for a connected graph G.

    (i) G is Eulerian.

    (ii) Every point of G has even degree.

    (iii) The set of edges of G can be partitioned into cycles.

20. Prove that if a connected graph G has exactly one spanning tree T, then G=T.

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