C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER I**

P18MMA103 – ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Let a_1, a_2 be constants and consider the equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are distinct roots of the characteristic polynomial p, where $p(r) = r^2 + a_1 r + a_2$, then the functions ϕ_1, ϕ_2 defined by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = e^{r_2 x}$ are solutions of $L(y) = 0$, If r_1 is a repeated root of p, then prove that the functions ϕ_1, ϕ_2 defined by $\phi_1(x) = e^{r_1 x}$, $\phi_2(x) = xe^{r_1 x}$ are solution of $L(y) = 0$.

(Or)

b) Let ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I, containing a point x_0, then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1,\phi_2)(x_0)$.

2. a) Let ϕ be any solution $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0$ on an interval I containing a point x_0. Then prove that for all x in I.

$\| \phi(x_0) \| e^{-K|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{K|x-x_0|}$, where $K = 1 + |a_1| + \ldots + |a_n|$.

(Or)

b) The correspondence which associates with each $L = a_0 D^n + a_1 D^{n-1} + \ldots + a_n$ its characteristics polynomial p given by $p(r) = a_0 r^n + a_1 r^{n-1} + \ldots + a_n$ is a one-to-one correspondence between all linear differential operators with constant coefficients and all polynomials. If L, M are associated with p, q respectively then prove that L + M is associated with p +q, ML is associated with pq and αL is associated with αp (α a constant).

3. a) State and prove uniqueness theorem for initial value problem for the homogeneous equation.

(Or)

b) Let $\phi_1 \ldots \phi_n$ be the n solutions of $L(y) = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1$, $\phi_i^{(j-1)}(x_0) = 0$, $j \neq i$. If ϕ is any solution of $L(y) = 0$ on I, prove that there are n constants $c_1 \ldots c_n$ such that $\phi = c_1 \phi_1 + \ldots + c_n \phi_n$.

4. a) Show that

(i) $J_{\alpha-1}(x) - J_{\alpha+1}(x) = 2J'_\alpha(x)$ (ii) $J_{\alpha-1}(x) + J_{\alpha+1}(x) = 2\alpha x^{-1} J'_\alpha(x)$.

(Or)

b) Compute the indicial polynomials, and their roots, for the following equation $x^2 y'' + (x + x^2) y' - y = 0$.

5. a) Verify the following equation is exact and solve them $y' = \frac{3x^2 - 2xy}{x^2 - 2y}$.

(Or)

b) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$ on I.

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SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Then prove that for all x in I.

   $\|\phi(x_0)\| e^{-K|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{K|x-x_0|}$

   where $\|\phi(x)\| = [\,|\phi(x)|^2 + |\phi^1(x)|^2]^{1/2}$, $K = 1 + |a_1| + |a_2|$.
7. Using the annihilator method, find a particular solution of $y'' - 4y = 3e^{2x} + 4e^{-x}$.
8. Obtain the solution of Legendre equation.
9. Derive Bessel functions of zero order of the second kind.
10. Let M, N be two real – valued functions which have continuous first partial derivatives on some rectangle $R : |x - x_0| \le a, |y - y_0| \le b$. Then prove that the equation $M(x, y) + N(x, y)\,y' = 0$ is exact in R if and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

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