C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER I**

P18MMA101 – ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Prove that N(a) is a subgroup of G.

(Or)

b) Prove with the usual notation that $n(k) = 1 + p + \cdots + p^{k-1}$.

2. a) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

(Or)

b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

3. a) Prove that $(f(x)\,g(x))' = f'(x)\,g(x) + f(x)\,g'(x)$, for any $f(x), g(x) \in F[x]$.

(Or)

b) If K is a field and if $\sigma_1, \ldots, \sigma_n$ are distinct automorphisms of K, then prove that it is impossible to find $a_1, \ldots, a_n$ not all 0, in K such that

$a_1\sigma_1(u) + a_2\sigma_2(u) + \cdots + a_n\sigma_n(u) = 0$ for all $u \in K$.

4. a) Prove that G is solvable if and only if $G^{(k)} = e$ for some integer k.

(Or)

b) Suppose that G is the internal direct product of $N_1, N_2, \ldots, N_k$. For $i \neq j$, prove that $N_i \cap N_j = (e)$ and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

5. a) If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$, where the $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.

(Or)

b) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. State and prove Sylow's theorem.
7. Prove that the element $a \in K$ is algebraic over F if and only if F (a) is a finite extension of F.
8. If F is of characteristic 0 and if a, b are algebraic over F, then there exist an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.
9. State and prove Wedderburn's theorem.
10. For each $i = 1, 2, \ldots, k$, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \cdots \oplus V_k$. Prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

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