C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER I**

P18EMA101 – FUZZY MATHEMATICS (ELECTIVE)

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Define fuzzy set and give an example.

(Or)

b) i) Define subsethood of a fuzzy set.

ii) Define standard union and intersection of two fuzzy sets.

2. a) State and prove third decomposition theorem.

(Or)

b) Prove that every fuzzy complement has at most one equilibrium.

3. a) Prove that the standard fuzzy intersection is the only idempotent t-norm.

(Or)

b) Given an involutive fuzzy complement C and an increasing generator g of C, the t-norm and t-conorm generated by g are dual with respect to C.

4. a) If $A(x) = \begin{cases} 0 & for\ x \le -1, x > 3 \\ \frac{x+1}{2} & for -1 < x \le 1 \\ \frac{3-x}{2} & for\ 1 < x \le 3 \end{cases}$ and $B(x) = \begin{cases} 0 & for\ x \le 1, x > 5 \\ \frac{x-1}{2} & for\ 1 < x \le 3 \\ \frac{5-x}{2} & for\ 3 < x \le 5 \end{cases}$, find $A + B$.

(Or)

b) Calculate the following

(i) $[0,1] + [5,6]$, (ii) $[0,1] - [5,6]$, (iii) $[1,2].[3,4]$ and (iv) $[4,10]/[1,2]$.

5. a) Explain the various types of binary fuzzy relations on a single set.

(Or)

b) The fuzzy relation R is defined on sets $X_1 = \{a,b,c\}$, $X_2 = \{s,t\}$, $X_3 = \{x,y\}$, $X_4 = \{i,j\}$ as follows:

$R(X_1,X_2,X_3,X_4) = 0.4/b,t,y,i + 0.6/a,s,x,i + 0.9/b,s,y,i + 1/b,s,y,j + 0.6/a,t,y,j + 0.2/c,s,y,i.$

Compute the projections $R_{1,2,4}$, $R_{1,3}$ and R_4.

SECTION - B (3 X 15=45 Marks)

Answer **ANY THREE** Questions.

6. a) A fuzzy set A on $\mathbb{R}$ is convex if and only if

$$A(\lambda x_1 + (1-\lambda)x_2) \ge \min[A(x_1), A(x_2)]$$

for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0,1]$.

b) Consider the fuzzy sets A, B and C defined on the interval [0,10] of real numbers by the membership grade functions $A(x) = \frac{x}{x+2}$, $B(x) = 2^{-x}$, $C(x) = \frac{1}{1+10(x-2)^2}$. Find $\bar{A}$, $\bar{B}$, $A \cup B$, $A \cap B$ and $A \cap \bar{C}$.

7. i) State and prove *first decomposition* theorem.

ii) Find α^A where the fuzzy set A given below and $\alpha \in [0,1]$

$$A(x) = \begin{cases} x-1 & for\ \ x \in [1,2] \\ 3-x & for\ \ x \in [2,3] \\ 0 & for\ \ otherwise \end{cases}$$

8. State and prove first characterization theorem of t-conorms.

9. Let A be fuzzy set defined on $\mathbb{R}$. Prove that A is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \emptyset$ such that

$$A(x) = \begin{cases} 1 & for\ x \in [a, b] \\ l(x) & for\ x \in (-\infty, a) \\ r(x) & for\ x \in (b, \infty) \end{cases}$$ where $l: (-\infty, a) \to [0,1]$ that is monotonically increasing, continuous from the right, and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; $r: (b, \infty) \to [0,1]$ that is monotonically decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (\omega_2, \infty)$.

10. For any fuzzy relation R on X^2, prove that $R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$ is the i-transitive closure of R.

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