C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER III**

P15MMA302 – TOPOLOGY

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Define the order topology, the product topology and the subspace topology.

(Or)

b) Let X be a space satisfying the T_1 axiom and let A be a subset of X. Prove that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.

2. a) State and prove the pasting lemma.

(Or)

b) State and prove the uniform limit theorem.

3. a) Define a connected space. Prove that the image of a connected space under a continuous map is connected.

(Or)

b) Define a locally connected space. Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

4. a) State and prove the tube lemma.

(Or)

b) State and prove the Lebesgue number lemma.

5. a) Let X be a topological space and let one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V} \subset U$.

(Or)

b) Prove that every metrizable space is normal.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Let A be a subset of the topological space X. (i) Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A. (ii) Supposing the topology of X is given by a basis, prove that $x \in \overline{A}$ if and only if every basis element B containing x intersects A. (iii) Prove that $\overline{A} = A \cup A'$ where A' is the set of all limit points of A.
7. Define the euclidean metric d and the square metric ρ on $\mathbb{R}^n$. Prove that the topologies on $\mathbb{R}^n$ induced by the euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.
8. (i) Prove that a finite Cartesian product of connected spaces is connected and (ii) State and prove the intermediate value theorem.
9. State and prove the following theorems: (i) Extreme value theorem and (ii) Uniform continuity theorem.
10. State and prove Urysohn lemma.

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