C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER III**

P15MMA301 – COMPLEX ANALYSIS - I

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) If the pricewise differentiable closed curve γ does not pass through the point 'a' then prove that the value of the integral $\int_\gamma dz / z\text{-}a$ is a multiple of $2\pi i$.

(Or)

b) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

2. a) Prove that a region Ω is simply connected if and only if $n(\gamma,a) = 0$ for all cycles γ in Ω and all points 'a' which do not belong to Ω.

(Or)

b) State and prove Rouche's theorem.

3. a) Derive Poisson's formula.

(Or)

b) State the Mean –value property and prove that the arithmetic mean of a harmonic function over concentric circles $|z| = \gamma$ is a linear function of $\log r$, i.e. $1/2\pi \int_{|z|=r} u\, d\theta = \propto \log r + \beta$, and if u is harmonic in a disk $\propto = 0$ and the arthimetic mean is constant.

4. a) State and prove Weierstrass's theorem.

(Or)

b) Derive Laurent series.

5. a) Derive Jensen's formula.

(Or)

b) Show that $\Gamma(1/6) = 2^{-1/3} (3/\pi)^{1/2} \Gamma(1/3)^2$.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Prove that following:

i) If f(z) is analytic and non-constant in a region Ω, then its absolute value $|f(z)|$ has no maximum in Ω.

ii) If f(z) is analytic for $|z| < 1$ and satisfies the conditions $|f(z)| \leqq 1$, $f(0)=0$, then $|f(z)| \leqq |z|$ and $|f'(0)| \leqq 1$. If $|f(z)| = |z|$ for some $z \neq 0$, or if $|f'(0)| = 1$, then $f(z) = cz$ with a constant c of absolute value 1.

7. If $p\,dx + q\,dy$ is locally exact in Ω, then prove that $\int_\gamma p\,dx + q\,dy = 0$ for every cycle $\gamma \sim 0$ in Ω.

8. Evaluate the following integrals by the method of residues.

a) $\int_0^{\pi/2} dx / (a + \sin^2 x)$, $|a| > 1$, b) $\int_0^\infty x^2 dx / (x^4+5x^2+6)$

9. State and prove Schwarz's theorem.

10. State and prove Hadamard's theorem.

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