C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),

MELVISHARAM - 632 509.

SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER I**

P15MMA102 – REAL ANALYSIS - I

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) Prove that every function of bounded variation is bounded.

(Or)

b) Let f be defined on [a , b],then prove that f is of bounded variation on [a , b], iff f can be expressed as the difference of two increasing function.

2. a) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a , b] then prove the $c_1 f + c_2 g \in R(\alpha)$ on [a , b] and $\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.

(Or)

b) State and prove Euler's Summation formula.

3. a) State and prove the second fundamental theorem of integral calculus

(Or)

b) State and prove the first mean value theorem for Riemann Stieltjes integral.

4. a) Define Cesaro summability and prove that if a series is convergent with sum S, then it is also (c, 1) summable with cesaro sum S.

(Or)

b) Let $\sum a_n$ be an absolutely convergent series having sum S, then prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum S.

5. a) Assume that $\lim_{n\to\infty} f_n = f$ on [a , b].

If $g \in R$ on on [a , b],define

$$h(x) = \int_a^x f(t)\, g(t)\, dt$$

$$h_n(x) = \int_a^x f_n(t)\, g(t)\, dt \quad \text{if } x \in [a,b].$$

then prove that h_n converges to h uniformly on [a,b]

(Or)

b) Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point C of S, then prove that the limit function f is also continuous at C.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Let f be of bounded variation on [a , b],and assume that $c \in (a , b)$, then prove f is of bounded variation on [a , c] and on [c , b] and $V_f(a,b) = V_f(a,c) + V_f(c,b)$.

7. Assume $f \in R(\alpha)$ on [a , b] and assume that α has a continuous derivative α' on [a , b].Then prove that the Riemann integral $\int_a^b f(x)\, \alpha'(x)\, dx$ exists and $\int_a^b f(x)\, d\alpha(x) = \int_a^b f(x)\, \alpha'(x)\, dx$.

8. Let $Q = \{(x, y) : a \le x \le b, c \le y \le d\}$. Assume that $\propto$ is of bounded variation on $[a,b]$, β is of bounded variation on $[c,d]$, and f is continuous on Q.

If $(x,y) \epsilon Q$, Define $\quad F(y) = \int_a^b f(x,y)\, d \propto (x)$

$$G(x) = \int_c^d f(x,y)\, d\beta(y).$$

Then prove that $F \epsilon R(\beta)$ on $[c, d]$ $G \epsilon R(\propto)$ on $[a, b]$ and

$$\int_c^d f(y) d\beta(y) = \int_a^b G(x)\, d \propto (x).$$

9. State and prove Dirichlet's test and Abel's test.

10. State and prove Weierstrass M-test.

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