C. ABDUL HAKEEM COLLEGE (AUTONOMOUS),
MELVISHARAM - 632 509.
SEMESTER EXAMINATIONS, NOVEMBER - 2018

M.Sc., MATHEMATICS **SEMESTER III**

P15EMA301 – PROBABILITY THEORY (ELECTIVE)

Time: Three Hours Maximum: 75 Marks

SECTION - A (5 X 6 = 30 Marks)

Answer **ALL** Questions.

1. a) State and Prove the theorem of absolute Probability.

(Or)

b) If X and Y are two independent random variables such that $f(x) = e^{-x}$, $x \geq 0$ and $f(x) = e^{-y}$, $y \geq 0$, find the probability distribution of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V are independent?

2. a) State and Prove Chebyshev's inequality.

(Or)

b) Prove that the equality $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(y = ax + b) = 1$ to hold.

3. a) Find the characteristic function, moments and the semi – invariants of a Poisson Distribution.

(Or)

b) Find the density of the random variable whose characteristic functions $\phi(t) = e^{-t^2/2}$, $-\infty < t < \infty$.

4. a) Let X_n be a binomial distribution with parameters n and p. If λ = np then prove that $\lim_{n \to \infty} P(X_n = r) = \frac{\lambda^r}{r!} e^{-\lambda}$.

(Or)

b) The random variable X has the distribution N (1,2). Find the probability that X is greater than 3 in absolute value.

5. a) Derive Kolmogorov Inequality.

(Or)

b) State and Prove Khintchine Weak law of large numbers.

SECTION - B (3 X 15 = 45 Marks)

Answer **ANY THREE** Questions.

6. Prove that the single value F(X) is a distribution function of a random variable x if and only if it is non – decreasing, continuous at least from the left and satisfying $F(-\infty) = 0$ and $F(+\infty) = 1$.
7. Let $(X_1, X_2, \dots X_n)$ be n – dimensional random variable. Obtain the regression hyper plane of second type of the random variable X_1 on the remaining variables $X_2, X_3, \dots X_n$. Also find the regression curve of the first type of the random variable X on Y for joint density function $f(x, y) = \frac{1}{3}(x + y)$ where $0 \leq x \leq 1, 0 \leq y \leq 2$.
8. State and Prove Levy's Uniqueness theorem.
9. Define Polya distribution and find its mean and variance.
10. State and Prove the Lindeberg – Levy Central theorem.

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